

$$1) \quad 5x_1 + 6x_2 + 12x_3 = -1$$

$$2x_1 - 3x_2 = -1$$

$$4x_1 + x_2 - x_3 = 6$$

$$A = \begin{bmatrix} 5 & 6 & 12 \\ 2 & -3 & 0 \\ 4 & 1 & -1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \times \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$l_{11} = a_{11} = 5$$

$$l_{21} = a_{21} = 2$$

$$l_{31} = a_{31} = 4$$

$$u_{12} = a_{12}/l_{11} = 6/5$$

$$u_{13} = a_{13}/l_{11} = 12/5$$

2ª coluna de L:

$$l_{22} = a_{22} - l_{21}u_{12} = -3 - 2 \times 6/5 = -27/5$$

$$l_{32} = a_{32} - l_{31}u_{12} = 1 - 4 \times 6/5 = -19/5$$

2ª linha de U:

$$u_{23} = (a_{23} - l_{21}u_{13})/l_{22} = (0 - 2 \times \frac{12}{5}) / (-\frac{27}{5}) = \frac{24}{27}$$

3ª coluna de L:

$$l_{33} = a_{33} - l_{31}u_{13} - l_{32}u_{23} = -1 - 4 \cdot 12/5 - (-19/5) \cdot \frac{24}{27} = -\frac{65}{9}$$

$$L = \begin{bmatrix} 5 & 0 & 0 \\ 2 & -27/5 & 0 \\ 4 & -19/5 & -65/9 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 6/5 & 12/5 \\ 0 & 1 & -24/27 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Ly = b$$

$$\begin{bmatrix} 5 & 0 & 0 \\ 2 & -27/5 & 0 \\ 4 & -19/5 & -65/9 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 6 \end{bmatrix}$$

$$y_1 = -1/5$$

$$2y_1 - 27/5 y_2 = -1$$

$$-\frac{2}{5} - \frac{27y_2}{5} = -1 \Rightarrow -27y_2 = -3$$

$$y_2 = 1/9$$

$$4y_1 - 19/5 y_2 - 65/9 y_3 = 6$$

$$-\frac{4}{5} - \frac{19}{5} \times \frac{1}{9} - \frac{65}{9} y_3 = 6 \Rightarrow -\frac{65}{9} y_3 = 6 + \frac{4}{5} + \frac{19}{45}$$

$$\frac{-65 \times 5}{45} y_3 = \frac{6 \times 45}{45} + \frac{4 \times 9}{45} + \frac{19}{45}$$

$$-325 y_3 = 270 + 36 + 19$$

$$y_3 = -1$$

$$Ux = y$$

$$\begin{bmatrix} 1 & 6/5 & 12/5 \\ 0 & 1 & -24/22 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1/5 \\ 1/9 \\ -1 \end{bmatrix}$$

$$x_3 = -1$$

$$x_2 - \frac{24}{22}x_3 = 1/9 \Rightarrow x_2 = \frac{1}{9} + \frac{24}{22} = \frac{3}{22} + \frac{24}{22}$$

$$x_2 = 1$$

$$x_1 + 6/5x_2 + 12/5x_3 = -1/5$$

$$x_1 = -1/5 - 6/5 + 12/5 = 5/5 = 1$$